

# Black Holes and Attractors in Supergravity

**Anna Ceresole<sup>a,b</sup> and Sergio Ferrara<sup>b,c</sup>**

<sup>a</sup> *INFN, Sezione di Torino, Via Pietro Giuria 1, 10125 Torino, Italy*  
*E-mail: ceresole@to.infn.it*

<sup>b</sup> *Physics Department, Theory Unit, CERN,*  
*CH -1211, Geneva 23, Switzerland*  
*E-mail: sergio.ferrara@cern.ch*

<sup>c</sup> *INFN - Laboratori Nazionali di Frascati,*  
*Via Enrico Fermi 40, I-00044 Frascati, Italy*

## ABSTRACT

We discuss some of the basic features of extremal black holes in four-dimensional extended supergravities. Firstly, all regular solutions display an attractor behavior for the scalar field evolution towards the black hole horizon. Secondly, they can be obtained by solving first order flow equations even when they are not supersymmetric, provided one identifies a suitable superpotential  $W$  which also gives the black hole entropy at the horizon and its ADM mass at spatial infinity. We focus on  $N=8$  supergravity and we review the basic role played by U-duality of the underlying supergravity in determining the attractors, their entropies, their masses and in classifying both regular and singular extremal black holes.

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# 1 Extremal Black Holes

In 1976, at the dawn of the  $N=1$  supersymmetric theory of gravity in four dimensions [1](now called “ $N=1$  supergravity”), when its  $N=2$  extension had just appeared, Murray Gell-Mann was the first to remark during a seminar at Caltech that, if higher  $N$  supergravity would have indeed existed, then there would be a bound such that  $N_{\max}=8$  would be the end of the story. Today, after 34 years, we are still struggling to understand this beautiful maximally extended theory[2], its connection with superstring and M-theory[3], its hypothetical perturbative finiteness and its non perturbative completion. This contribution focusses on the black holes that arise in  $N=8$  supergravity in four dimensions[4], which have been recently claimed to play a possible key role in relation to string theory and to the issue of perturbative finiteness of  $N=8$  supergravity[5].

Black holes, one of the most interesting outcome of General Relativity, are the typical probes of the quantum regime of any fundamental theory of gravity and as such, they are naturally investigated within the framework of superstring and M-theory. As a first approximation, they can be proficiently studied as classical solutions of the underlying extended supergravities, which arise upon compactification in the effective field theory limit. Once the background geometry of the  $d$ -dimensional spacetime and the number  $N$  of supersymmetry charges have been selected, all the important features of a given solution are encoded into the electric magnetic-duality group  $G$  acting on the vector fields  $A^\Lambda$ , and in the geometric properties of the moduli space  $G/H$  parametrized by the scalar fields  $\phi^i$ [6]. When the electric and magnetic charges are quantized, the group  $G$  becomes the U-duality group which is known to dictate the string dynamics in various dimensions[7, 8].

The thermodynamical properties of black holes can be obtained from quantum mechanical attributes that are their (ADM) mass, charge, spin and scalar charges (see for instance [9, 10, 11, 12]). Unlike Schwarzschild black holes, charged (Reissner-Nordstrom) and/or spin (Kerr-Newman) black holes can be *extremal*, i.e with vanishing temperature for non-zero entropy, in which case their event horizon and Cauchy horizons coincide. In formulae, the extremality parameter is given by

$$c = 2ST = \frac{1}{2}(r_+ - r_-) \rightarrow 0, \quad (1.1)$$

where  $c$  measures the surface gravity and  $S = \log \mathcal{N}$  is the black hole entropy which counts the number  $\mathcal{N}$  of microstates. In supergravity, it is given by the Bekenstein-Hawking area formula

$$S_{BH} = \frac{1}{2}A_H = \pi R_H^2, \quad (1.2)$$

where  $R_H$  is the effective radius of a sphere encircling the horizon. For extremal charged black holes,  $R_H$  must respect the symmetries of the theory and in particular it must depend only on the electric and magnetic charges and not on the scalar field values [6].

Therefore, also the entropy will depend only on the charges and it will take particular expressions depending on the duality symmetries of the given model.

The lack of dependence on scalar fields of the entropy can be viewed as a sort of “no hair” theorem [13] and reflects the fact that, under certain conditions, extremal black holes in N-extended supergravities enjoy a remarkable property: the scalar field trajectories  $\phi^i(\tau)$ , in terms of the radial evolution parameter  $\tau = -1/\rho$  from asymptotic infinity ( $\tau \rightarrow 0$ ) to the horizon ( $\tau \rightarrow -\infty$ ), behave as dynamical systems, *i. e.* independently on their initial values  $\phi_0^i = \phi^i(\tau)|_{\tau \rightarrow 0}$ . The scalars evolve to a common value at the horizon,  $\phi_H^i = \phi_H^i(Q)$ , which they reach with zero velocity ( $\dot{\phi}^i \rightarrow 0$ ), and where they entirely depend on the electric-magnetic charge vector  $Q$  of the asymptotic configuration. This attractive feature is called *Attractor Mechanism* [10, 11], and the attractor fixed points can be obtained as extrema of a suitable effective potential. Another important feature of extremal black holes is that their horizon geometry of spacetime is universal, and in four dimensions it is given by the  $AdS_2 \times S_2$  Bertotti-Robinson metric. This is a particular case of the geometry of black p-branes in D-dimensions and it is an instance of the AdS/CFT correspondence relating the gauge theory on the boundary to bulk supergravity[14]. Thus, extremal black holes behave as solitons interpolating between maximally symmetric geometries of (super)spacetime: Minkowski for  $\tau \rightarrow 0$  and the conformally flat metric for  $\tau \rightarrow -\infty$  [15, 16].

It is remarkable that the common radius of  $AdS_2 \times S_2$  and therefore the entropy, can be actually computed using the electric-magnetic duality of the underlying supergravity theory.

For Reissner-Nordstrom black holes, with electric charge  $q$  and magnetic charge  $p$ , the entropy is  $S \sim p^2 + q^2$  due to the  $U(1)$  symmetry that rotates the  $p$  and  $q$  charges into each other. For more complicated objects where scalar fields are present, such as the Axion-Dilaton black hole having two  $U(1)$  gauge fields and charges  $(q_1, p_1)$  and  $(q_2, p_2)$ , the entropy becomes  $S \sim |p_1 q_2 - p_2 q_1|$  and it is invariant under  $SL(2) \times SO(2)$  [35]. The appearance of a non compact symmetry group is a quite general signature of the presence of scalar fields. In  $N = 4$  supergravity, one has  $S \sim |p^2 q^2 - (p \cdot q)^2|^{1/2}$ , with  $SL(2) \times SO(6, n)$  symmetry[34]. In the maximal  $N = 8$  case, the dyonic charge vector  $Q^a$  transforms in the fundamental **56** representation of  $E_{7(7)}$  (since there are 28 vector fields yielding 28 electric and 28 magnetic charges) while the scalar fields span the 70-dimensional scalar manifold  $G/H = E_{7(7)}/SU(8)$ . It turns out that the entropy for regular black holes is proportional to  $\sqrt{|I_4|}$  where  $I_4$  is the quartic invariant of the **56** representation of  $E_{7(7)}$ ,  $I_4 = T_{abcd} Q^a Q^b Q^c Q^d$ . When  $I_4 = 0$ , the black hole is singular, with vanishing horizon area, and there can be 1/8, 1/4 or 1/2 supersymmetry preserved [31, 17, 18].

In the last few years it has become clear that the scalar field dynamics for the extremal black holes can be entirely encoded into a real “superpotential” function  $W(\phi^i, Q)$  for

which  $\phi_H^i$  is a critical point in the moduli space of the theory:  $\partial W / \partial \phi^i|_{\phi^i=\phi_H^i} = 0$ .

For BPS (supersymmetric) configurations,  $W = |z_h| = \rho_h$  where  $\rho_h$  is the modulus of the highest among the skew eigenvalues of the central charge matrix  $Z_{AB} = -Z_{BA}$  of the supersymmetry algebra,

$$\{\mathcal{Q}_{\alpha A}, \mathcal{Q}_{\beta B}\} = \epsilon_{\alpha\beta} Z_{AB}(\phi_0^i, Q) \quad (1.3)$$

$$\{\mathcal{Q}_{\alpha A}, \mathcal{Q}_{\beta}^B\} = \sigma_{\alpha\beta}^\mu P_\mu \delta_A^B. \quad (1.4)$$

Depending on how many of these skew eigenvalues are coincident, one has various degrees of preserved supersymmetry. For the N=8 theory, one can have four different cases ranging from 1/8, to 1/4, 1/2 and zero preserved supersymmetries. In the original setup of N=2 extremal black holes in D=4,  $Z_{AB} = \epsilon_{AB} Z$  and such a superpotential function for supersymmetric configurations was to be identified with the modulus of the N=2 central charge  $Z$ , appearing in the ordinary BPS equations [10, 11]. Remarkably, such a function  $W$  can be shown to exist also for non supersymmetric configurations, in which case it is called the “fake superpotential” [19] because of the similarity with the set up of “fake supergravities” [20]. When the attractors are regular, the  $W$  function has a minimum for  $\phi^i = \phi_H^i$ , and its horizon value gives the entropy of the configuration

$$S = \frac{1}{4} A_H = \pi W_H^2(Q) = \pi W_{\text{crit}}^2(\phi_h^i(Q), Q) \quad (1.5)$$

according to the Bekenstein-Hawking formula. However, if  $W$  has a runaway behavior in moduli space,  $\phi_H \rightarrow \infty$   $W \rightarrow 0$  (which is not acceptable in D=4), the corresponding black hole solutions are singular. Then the scalar fields are never stabilized within the boundaries of moduli space, there are no attractors and the entropy of the extremal configuration vanishes.

In order to describe a static, spherically symmetric extremal black hole background in the extremal case,  $c = 2ST = 0$ , the metric ansatz reads [11]

$$ds^2 = -e^{2U} dt^2 + e^{-2U} \frac{d\tau^2}{\tau^4} + \frac{1}{\tau^2} (d\theta^2 + \sin^2 \theta d\phi^2). \quad (1.6)$$

with the field strength  $F_{\mu\nu}^\Lambda$  for  $n_v$  vectors ( $\Lambda = 1, \dots, n_v$ ) and its dual  $G_{\Lambda\mu\nu} = \frac{\delta \mathcal{L}}{\delta F_{\mu\nu}^\Lambda}$  given by

$$F = e^{2U} C \mathcal{M}(\phi^i) Q dt \wedge d\tau + Q \sin \theta d\theta \wedge d\phi \quad (1.7)$$

$$F = \begin{pmatrix} F_{\mu\nu}^\Lambda \\ G_{\Lambda\mu\nu} \end{pmatrix} \frac{dx^\mu dx^\nu}{2}. \quad (1.8)$$

Electric and magnetic charges are defined by

$$q_\Lambda = \frac{1}{4\pi} \int_{s_2} G_\Lambda, \quad p^\Lambda = \frac{1}{4\pi} \int_{S_2} F_\Lambda. \quad (1.9)$$

$\mathcal{M}(\phi^i)$  is a  $2n_v \times 2n_v$  real symmetric  $Sp(2n_v, R)$  matrix, satisfying  $\mathcal{M}C\mathcal{M} = C$ ,

$$\mathcal{M}(\phi^i) = \begin{pmatrix} I + RI^{-1}R & -RI^{-1} \\ -I^{-1}R & I^{-1} \end{pmatrix} \quad (1.10)$$

where  $I = Im \mathcal{N}_{\Lambda\Sigma}$ ,  $R = Re \mathcal{N}_{\Lambda\Sigma}$ , the vector kinetic matrix  $\mathcal{N}_{\Lambda\Sigma}$  depends on the scalar fields and enters the 4D lagrangian

$$\mathcal{L} = -\frac{R}{2} + \frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j + I_{\Lambda\Sigma}F^\Lambda \wedge^* F^\Sigma + R_{\Lambda\Sigma}F^\Lambda \wedge F^\Sigma. \quad (1.11)$$

The black hole effective potential [10] is given by

$$V_{BH} = -\frac{1}{2}Q^T \mathcal{M} Q = \frac{1}{2}Z_{AB}\bar{Z}^{AB} = \sum_i \rho_i^2, \quad (1.12)$$

where A,B are SU(8) indices [12]. In the last expression,  $\rho_i$  are the moduli of the skew eigenvalues  $z_i$  of the central charge matrix  $Z_{AB}$ . It arises upon reducing the general 4D (or higher D) lagrangian to the one-dimensional almost geodesic action describing the radial evolution of the  $n+1$  scalar fields  $(U(\tau), \phi^i(\tau))$ :

$$S = \int \mathcal{L} d\tau = \int (\dot{U} + g_{i\bar{j}}\dot{\phi}^i\dot{\phi}^{\bar{j}} + e^{2U}V_{BH}(\phi(\tau), p, q))d\tau. \quad (1.13)$$

In order to have the same equations of motion of the original theory, the action must be complemented with the Hamiltonian constraint (in the extremal case) [11]

$$\dot{U}^2 + g_{ij}\dot{\phi}^i\dot{\phi}^j - e^{2U}V_{BH}(\phi(\tau), p, q) = 0. \quad (1.14)$$

The black hole effective potential can be written in terms of the superpotential  $W(\phi)$  as

$$V_{BH} = W^2 + 2g^{ij}\partial_i W \partial_j W. \quad (1.15)$$

This formula can be viewed as a differential equation defining W for a given black hole effective potential  $V_{BH}$ , and it can lead to multiple choices: only one of those will corresponds to BPS solutions, while a different one will be associated to non BPS ones. In both cases, W allows to rewrite the ordinary second order supergravity equations of motion

$$\frac{d^2 U}{d\tau^2} = e^{2U} V_{BH} \quad (1.16)$$

$$\frac{d^2 \phi^i}{d\tau^2} = g^{i\bar{j}} \frac{\partial V_{BH}}{\partial \phi_{\bar{j}}} e^{2U}, \quad (1.17)$$

as first order flow equations, defining the radial evolution of the scalar fields  $\phi^i$  and the warp factor  $U$  from asymptotic infinity towards the black hole horizon [19] :

$$U' = -e^U W, \quad \phi'^i = -2e^U g^{ij}\partial_j W. \quad (1.18)$$

The important point is that, at the prize of finding a suitable fake superpotential  $W$ , one only has to deal with these first order flow equations even for non supersymmetric solutions, where one does not have Killing spinor equations [19, 23].

Beside the horizon entropy  $S_{BH} = \pi W_H^2$  and the first order flows, the value at radial infinity of the superpotential  $W$  also encodes other basic property of the extremal black hole, which are its  $ADM$  mass, given by

$$M_{ADM}(\phi_0^i, Q) = \dot{U}(\tau = 0) = W(\phi_0, Q) \quad (1.19)$$

and the scalar charges at infinity

$$\Sigma^i = \dot{\phi}^i(\tau = 0) = 2g^{ij}(\phi_0) \frac{\partial W}{\partial \phi^i}(\phi_0, Q) \quad (1.20)$$

A finite horizon area demands that  $\dot{\phi}^i(\tau = -\infty) = 0$  and thus  $\frac{\partial W}{\partial \phi^i}|_{\phi_H^i} = 0$  and

$$\lim_{\tau \rightarrow -\infty} e^{-2U} = R_H^2 \tau^2, R_H^2 = W_H^2(\phi_H, Q) \quad (1.21)$$

and  $R_H^2(Q) = |I_4(Q)|^{1/2}$ , so that indeed the effective radius is given in terms of the Cartan quartic invariant of the 56 of  $E_{7(7)}$ .

By using the asymptotic behaviour of the warp factor at spacial infinity  $\tau \rightarrow 0$ , one has  $U \rightarrow -M_{ADM}\tau$  and at the horizon  $\tau \rightarrow \infty$

$$e^{-2U} \rightarrow \frac{1}{4\pi} A_H \tau^2 \quad (1.22)$$

Thus, one gets model independent expressions for ADM mass, scalar charges and entropy as a function of  $W$

$$M_{ADM}^2 = W_{\tau \rightarrow 0}^2 = W_\infty^2 \quad (1.23)$$

$$\Sigma^i = -2g^{ij} \frac{\partial W}{\partial \phi^j} |_{\tau \rightarrow 0} = -2g^{ij} \frac{\partial W}{\partial \phi^j} |_\infty \quad (1.24)$$

$$S = \frac{1}{4} A_H = \pi W^2 |_{\tau \rightarrow -\infty} = \pi W^2 |_H \quad (1.25)$$

The horizon value of the scalar fields  $\phi_H^i$  is a critical point for  $W$  in the moduli space of the theory,  $\partial W / \partial \phi^i |_{\phi^i = \phi_H^i} = 0$ . It follows that for  $\tau \rightarrow 0$  (radial infinity)

$$M_{ADM}^2 = V_{BH}(\phi_\infty, p, q) - \frac{1}{2} g_{ij} \Sigma^i \Sigma^j \quad (1.26)$$

while for  $\tau \rightarrow -\infty$  (horizon)

$$S_{BH} = \frac{1}{4} A_H = \pi V_{BH}(\phi_H, p, q) = \pi V_{BH} |_{\phi_H} \quad (1.27)$$

showing *attractor behaviour* [10]. For BPS states

$$S_{BH} = \pi |z_h|_H^2 \quad M_{ADM}^2 = |z_h|_\infty^2, \quad (1.28)$$

where  $z_h$  is the skew eigenvalue of the central charge  $Z_{AB}$  with the highest modulus. Quite generally, one can prove the validity of the BPS bound along the flow

$$M_{ADM} \geq |z_i|. \quad (1.29)$$

At the horizon

$$\dot{\phi}^i|_{\tau \rightarrow -\infty} = 0 \rightarrow \partial_i W_H = 0 \quad (W_H \leftrightarrow W_{crit}). \quad (1.30)$$

For  $N \geq 2$ , the fake superpotential for the non-BPS branch[21, 22] has been computed for wide classes of models [19, 24, 23, 25, 26, 27, 28, 29, 38], based on symmetric geometries of moduli spaces, using as a tool the U-duality symmetry of the underlying supergravity. A universal procedure for its construction in  $N = 2$  special geometries has been established [26, 27], which generalizes the results obtained for the *stu* model[37]. This universal procedure can be also applied to the  $N = 8$  theory using the fact that the *stu* model is both a subsector of  $N = 8$  and a model in  $N = 2$  where we know how to describe  $W$  in terms of invariants. The results of [27] agree with the outcome of studies of black hole evolution with the method of time-like reduction to 3 dimensions in [28]. However, in both contexts, the expression for  $W$  is only given implicitly, as solution of a sixth order polynomial having for coefficients some  $SU(8)$  invariant functions composed out of the  $N = 8$  central charge.

## 2 Attractors and Duality orbits in N=8 supergravity

As already noticed above, U-duality of the underlying extended supergravity dictates many important features [7]. Another important point is that the absolute Cartan invariant  $I_4$  of the 56 dimensional representation of  $E_{7(7)}$  and its derivatives allow to specify the supersymmetric features of a given solution and to classify the U-duality (continuous) orbits for the charge vector  $Q$  [30, 32, 33].

The area of the horizon for both regular 1/8-BPS and non-BPS attractors, is proportional to the square of the absolute Cartan quartic invariant [17]

$$I_4 = Tr(Z\bar{Z})^2 - \frac{1}{4}(Tr Z\bar{Z})^2 + 4(Pf Z + Pf \bar{Z}). \quad (2.1)$$

This expression, involving the traces of the matrix  $Z_{AB}\bar{Z}^{BC}$  and of its square as well as the real part of the Pfaffian of  $Z_{AB}(\phi, Q)$ ,

$$Pf Z = \frac{1}{2^4 4!} \epsilon^{abcdefgh} Z_{AB} Z_{CD} Z_{EF} Z_{GH} \quad (2.2)$$

actually depends only on the electric and magnetic charges  $Q = (p, q)$  and not on the scalar fields. Moreover, for fixed values of  $I_4$  in d=4 and of an analogous cubic invariant

$I_3$  in d=5, charge vectors  $Q$  for supergravities on symmetric spaces describe related orbits and attractors [40, 41].

By an  $SU(8)$  transformation, one can reach the canonical bases where the antisymmetric central charge matrix is (skew)diagonalized and takes the form

$$Z_{AB} = \begin{pmatrix} \rho_0 & & & \\ & \rho_1 & & \\ & & \rho_2 & \\ & & & \rho_3 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} e^{i\varphi/4} \quad (2.3)$$

in terms of its skew eigenvalues  $z_i = \rho_i e^{i\varphi/4}$ ,  $i = 0, 1, 2, 3$  which amount to 5 independent parameters. The attractor equation becomes

$$\partial_i V = 0 \rightarrow z_i z_j + z_k^* z_l^* = 0, (i \neq j \neq k \neq l) \quad (2.4)$$

which yields two solutions

$$\text{a) } z_h \neq 0, z_{i \neq h} = 0$$

$$\text{b) } |z_i| = \rho, \arg Pf Z = \pi$$

Considering the quartic invariant in this basis,

$$I_4 = [(\rho_0 + \rho_1)^2 - (\rho_2 + \rho_3)^2][(\rho_0 - \rho_1)^2 \quad (2.5)$$

$$- (\rho_2 - \rho_3)^2] + 8\rho_0\rho_1\rho_2\rho_3(\cos \phi - 1) \quad (2.6)$$

$$\rho_0 \geq \rho_1 \geq \rho_2 \geq \rho_3 \quad (2.7)$$

one finds that there are two disjoint regular orbits corresponding to the a) and b) attractor solutions:

- i) 1/8 BPS: with  $I_4 > 0$ , for  $\rho_0 \neq 0$  and  $\rho_1 = \rho_2 = \rho_3 = 0$ . The duality orbit is  $E_{7(7)}/E_{6(2)}$ .
  - ii) non BPS: with  $I_4 < 0$ , for  $|z_i| = \rho$  and  $\arg Pf Z = \pi$ . The duality orbit is  $E_{7(7)}/E_{6(6)}$ .
- There are three additional “small” orbits, preserving respectively 1/8, 1/4 and 1/2 supersymmetry, which are singular, having  $I_4 = 0$  and thus zero horizon area.

The entropy of the two regular branches is given by

$$S_{1/8\text{BPS}} = \pi V_{BH}|_H = \pi \rho_0^2|_H, \quad (2.8)$$

$$S_{\text{nonBPS}} = \pi V_{BH}|_H = \pi (2\rho_0)^2|_H, \quad (2.9)$$

where  $\rho_0|_H$  is the fixed point value of the field dependent modulus of the highest eigenvalue  $\rho_0$ . A similar classification has recently been achieved also in the  $N = 2$  case, where the superpotential  $W$  was computed for large and small orbits [44]. The new feature in that case is that singular orbits for  $N=2$  black holes can also be non-BPS and that  $W$  can always be determined by simple radicals.



An interesting outcome of the study of black holes in  $N=8$  four dimensional supergravity is related to some recent results [68], which point to a possible key role played by black holes with singular geometry in relation to String Theory.  $N = 8$  supergravity in  $D = 4$  can be obtained by dimensional reduction of  $D=11$ ,  $N=1$  supergravity (M-theory) on a 7-torus  $T^7$ , or compactifying Type-II string theory on  $T^6$ . In [67] it has been recently observed that in the process of compactification, one is left not only with the 256 massless states of the  $N=8$ ,  $D=4$  supergravity, but also with an infinite tower of stringy elementary states with arbitrarily small mass, making the decoupling impossible. On the other hand, since the degeneracies of BPS states in string and M-theory are counted by U-duality invariant formulas, it is interesting to inquire whether one can consistently decouple these extra massless states without violating the U-duality invariance of the degeneracy formula. If one can't decouple the extra states without breaking U-duality, then this would suggest that one may be able to disprove the conjecture of UV finiteness of the perturbative  $N=8$  supergravity theory. From the 4D point of view, these additional massless states appear to correspond to classical black hole solutions carrying charges which lead to light-like orbits with  $I_4 = 0$  [68], and that  $N = 8$  black holes with vanishing area of the horizon can be interpreted as dynamically reduced black branes with regular geometry. For example,  $1/8$  BPS black holes with  $I_4 = 0$  correspond to reduced (wrapped) black holes (black strings) at  $D=5$  (with three charges  $q_1, q_2, q_3$ ,  $J_{MAX} = 7/2$ ). Conversely,  $1/4$  BPS black holes with  $\partial I_4 = 0$  correspond to reduced and wrapped black strings at  $D = 6$  (two charges  $q_1$  and  $q_2$ ,  $J_{MAX} = 3$ ). Finally,  $1/2$  BPS black holes with  $\partial^2 I_4 = 0$  correspond to KK states i.e. massless particles in higher dimensions  $J_{MAX} = 2$ . If only regular black holes, with  $AdS_2 \times S_2$  horizon geometries are to be retained in a consistent theory of gravity, one may not need to open up extra dimensions[67] : whether this feature could hint to a possible finite theory of  $D=4$ ,  $N = 8$  supergravity [5] without the need of string theory, is a question that is presently under close investigation.

### 3 Moduli spaces of attractors and flat directions

It is a property of  $N=2$  supergravity that the geometry of black holes, at least in the Einsteinian approximation, does not depend on the hypermultiplet moduli space[10]. This implies that attractive solutions exist for arbitrary v.e.v's of the hypermultiplet scalars. This phenomenon generalizes to the non-BPS  $N=2$  regular black holes as well as to small black holes, for which there is no attractor behaviour [42, 43, 44]. In fact, it has been realized that, in terms of the fake superpotential  $W$  (or, for BPS solutions, in terms of the  $Z$  central charge), such “flat directions” are flat directions for  $W$  along the complete flow and therefore the ADM mass as well. For  $N > 2$ , flat directions exist even for BPS regular attractive (large) black holes and for singular solutions. In this latter case, the moduli space of flat directions is usually bigger, and in fact for the doubly critical orbits

the W function just depends on one real modulus having the simple interpretation as the radius dependence of the Kaluza-Klein mass [45, 46]. Let us confine again the discussion to the N=8 case. The five charge orbits are as follows [31]

orbit	susy	coset	moduli space
$I_4 > 0$	1/8BPS	$\frac{E_{7(7)}}{E_{6(2)}}$	$\frac{E_{6(2)}}{SU(6) \times SU(2)}$
$I_4 < 0$	nonBPS	$\frac{E_{7(7)}}{E_{6(6)}}$	$\frac{E_{6(6)}}{USp(8)}$
$I_4 = 0, \partial I_4 \neq 0$	1/8BPS	$\frac{E_{7(7)}}{F_{4(4)} \odot T_{26}}$	$\frac{F_{4(4)}}{USp(6) \times USp(2)} \odot T_{26}$
$I_4 = 0, \partial I_4 = 0$	1/4BPS	$\frac{E_{7(7)}}{O(6,5) \odot (T_{32} + T_1)}$	$\frac{O(6,5)}{O(6) \times O(5)} \odot (T_{32} + T_1)$
$I_4 = 0, \partial I_4 = 0, \partial^2 I_4 = 0$	1/2BPS	$\frac{E_{7(7)}}{E_{6(6)} \odot T_{27}}$	$\frac{E_{6(6)}}{USp(8)} \odot T_{27}$

(3.1)

The last column refers to the moduli space of flat directions and is given by the stabilizer of the coset orbit divided by its maximal compact subgroup. One observes that small lightlike black holes, having only the condition  $I_4 = 0$ , apart from  $T_{26}$  translations, have the same moduli space as the 1/8 BPS large black hole in D=5 [42].

Remarkably, the 1/2 BPS black holes, apart from  $T_{27}$  additional translations, have the same moduli space as large non-BPS black holes and both are the 5D N=8 supergravity moduli space. This is due to the fact that both these configurations can be obtained without genuine 5D black hole charges but turning on the NUT charge and angular momentum, without breaking the  $E_{6(6)}$  symmetry of the 5D theory [24, 40, 41].

## 4 Conclusions and Other recent developments

Since the attractor mechanism was observed [10], there has been a lively activity around the study of black holes in connection with string theory and M-theory. To begin with, an intriguing connection was conjectured between the central charge and the topological supergravity partition function at the attractor point [48]. A further important development has been the computation of the entropy through microstate counting, reproducing the Bekenstein-Hawking formula in the regime of large charges [47].

Then, quantum corrections to the entropy formulae and to the attractor equations were computed in superstrings and supergravity with N=2 supersymmetry [51, 49, 52, 50]. Microstate counting for black holes was considered in many papers, including the case of small black holes for which the area formula does not apply, even in the classical regime [53, 55, 54]. Several aspects concerning multicenter black holes, black hole deconstructions, split attractor flows and walls of marginal stability have been thoroughly investigated [56, 57, 58, 59].

The connection between 5D and 4D has been explored under different perspectives, as for instance in [66, 40] among many other contributions. Three dimensional time reduction has also been shown to be a powerful tool for black hole classification, duality orbits and possible quantization of the geodesic flows [60, 61, 62].

As a final note, we would like to mention another more speculative aspect of black hole physics, namely the mathematical connection between black hole physics and Quantum Information Theory, in particular the relation between black hole entropy and qubits multipartite entanglement [64].

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